

Antibunching photons in a cavity coupled to an optomechanical system

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We study the photon statistics of a cavity linearly coupled to an optomechanical system via second order correlation functions. Our calculations show that the cavity can exhibit strong photon antibunching even when optomechanical interaction in the optomechanical system is weak. The cooperation between the weak optomechanical interaction and the destructive interference between different paths for two-photon excitation leads to the efficient antibunching effect. Compared with the standard optomechanical system, the coupling between a cavity and an optomechanical system provides a method to relax the condition for obtaining single photon by optomechanical interaction.

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I. INTRODUCTION

Single-photon sources are of considerable interest for important applications in quantum information processing (see, e.g., review [1] and references therein), for example, key distribution in quantum cryptography [2]. Initially, single-photons are achieved by attenuated laser pulses with an average number of photons per pulse smaller than one. While the emitting photons show bunching statistics, and this may result in low transmission rates or information leakage in quantum key distribution. Clearly, single-photon emitter should deliver one, and only one photon at a time, in other words, the emitting photons should show antibunching effect. Various mechanisms have been employed to arouse antibunching effect, and one of them is to utilize photon blockade, which is a kind of quantum optical effect of preventing subsequent photons from resonantly entering the cavity [3]. Many theoretical proposals have been made for generating photon blockade, such as, optical cavity with four-level atomic ensemble [3], quantum well exciton in a planar microcavity [4], coupled nonlinear cavities [5–7], and optomechanical systems [8, 9]. To date, photon blockade has been observed in both optical cavities [10, 11] and superconducting circuits [12, 13].

Optomechanical system is attracting more and more attention because of many great experimental achievements in recent years (see, reviews [14, 15]). Sideband cooling of a mechanical oscillator into its ground state paves the way for putting mechanical oscillators into quantum mechanics [16–30]. The optical response of optomechanical systems to signal field can be modified by driven the cavity with a strong laser field, leading to effects such as electromagnetically induced transparency (EIT) [31–35] and normal-mode splitting [31, 36]. Normal-mode splitting is the evidence for entering the strong optomechanical coupling regime.

In Refs. [8, 9], the statistical properties and blockade of photons in optomechanical systems have been theoretically studied. These studies show that the nonlinear quantum optics at single-photon level using optomechanical systems becomes possible under strong optomechanical coupling condition. Thus the optomechanical system can act as a single-photon source. Moreover, we showed that photon induced

tunneling can occur in optomechanical systems [37]. Recently, it was shown that the nonlinear interactions in two coupled optomechanical systems can be significantly enhanced for mechanical frequencies nearly resonant to the optical level splitting [38, 39]. However, the photon blockade effects still only appear in the strong coupling regime, which is beyond the reach of most experiments in the single photon regime. Thus there is a question whether single-photon states can be generated using weak optomechanical interaction.

Recently, Liew and Savona analyzed two coupled nonlinear cavities [6], they found that the single-photon statistics can exhibit strong antibunching in such coupled-mode systems with weak Kerr nonlinearity compared to the single-mode case. Later on, such strong antibunching is attributed to the destructive quantum interference effect [7], and authors further extended their theory to two coupled-cavities [7] with a two-level quantum emitter, which is embedded in one of cavities. They theoretically demonstrated that perfect photon antibunching can be obtained even for single-atom cooperativity on the order of or smaller than unity. These studies [6, 7] have opened up a door towards nonlinear quantum optics at single-photon level using weak nonlinear coupling.

Motivated by studies in Refs. [6, 7] and also recent theoretical and experimental progress in coupled-cavity array and optomechanical systems, we now study the properties of the light field inside a cavity, which is coupled to an optomechanical system. The paper is organized as follows. In Sec. II, the model Hamiltonian is introduced. In Sec. III, quantum Langevin equations of cavity fields and mechanical mode are given, and the statistical properties of the cavity fields are studied using the second-order correlation function under the semiclassical approximation in Sec. IV. In Sec. V, we further analyze the second-order correlation function by numerical simulation via the master equation, and compare those with the results of the semiclassical approximation. Finally, summary and conclusions are given in Sec. VI.

II. MODEL

As schematically shown in Fig. 1, the system consists of two coupled cavities (A and B) with the coupling constant

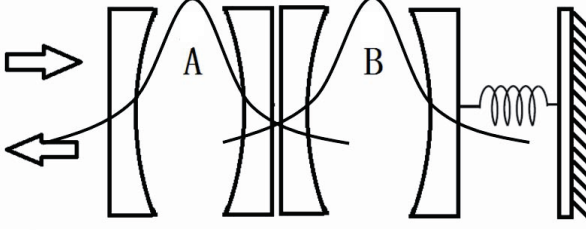


FIG. 1: Schematic diagram for an optical cavity (cavity A) coupled to an optomechanical system which consists of an optical cavity (cavity B) with an oscillating mirror at one end. Cavity A is driven by a weak coherent laser field.

g_0 . The cavity can be a transmission line resonator, a toroidal microresonator, a cavity with two mirrors, or a defect cavity in photonic crystal. Without loss of generality and for simplicity, we will focus on the cavity with two mirrors. Cavity A is driven by a weak probe field with frequency ω_c , and cavity B consists of an oscillating mirror at one end, modeled as a quantum mechanical harmonic oscillator. In other words, we study a coupled system, which consists of a driven cavity and an optomechanical system. The Hamiltonian of the whole system in the rotating wave approximation is given as

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b + \hbar\omega_m c^\dagger c + \hbar g_0 (a^\dagger b + b^\dagger a) + \hbar G b^\dagger b (c^\dagger + c) + i\hbar\varepsilon_c (a^\dagger e^{-i\omega_c t} - a e^{i\omega_c t}), \quad (1)$$

where a (a^\dagger) is the annihilation (creation) operator for the light mode of the cavity A with frequency ω_a , b (b^\dagger) is the annihilation (creation) operator for the light mode of the cavity B with frequency ω_b , and c (c^\dagger) is phonon annihilation (creation) operator of the mechanically vibrational mode for the mirror with frequency ω_m . The parameter G denotes the coupling strength between the cavity B and the oscillating mirror, and ε_c presents the coupling strength between the driven field and cavity field inside the cavity A. As $\omega_a \approx \omega_b \gg \omega_m, g_0$, we have dropped the rapidly varying terms (ab and $a^\dagger b^\dagger$) corresponding to the rotating wave approximation.

To remove the time-dependent factor, let us transform the Hamiltonian in Eq. (1) into the rotating reference frame through a unitary operator $R(t) = \exp[-i\omega_c t(a^\dagger a + b^\dagger b)]$, and thus the Hamiltonian in Eq. (1) becomes

$$\tilde{H} = \hbar\Delta_a a^\dagger a + \hbar\Delta_b b^\dagger b + \hbar\omega_m c^\dagger c + \hbar g_0 (a^\dagger b + b^\dagger a) + \hbar G b^\dagger b (c^\dagger + c) + i\hbar\varepsilon_c (a^\dagger - a), \quad (2)$$

where $\Delta_a = \omega_a - \omega_c$ and $\Delta_b = \omega_b - \omega_c$ are the detunings of the frequencies of cavity fields from that of the driving field.

III. LANGEVIN EQUATIONS AND SEMICLASSICAL APPROXIMATION

The dynamics of the cavity fields and mechanical oscillator can be described by quantum Langevin equations. By con-

sidering the dissipation and fluctuation of the light fields of two cavities and mechanical mode, we can write out a set of nonlinear quantum Langevin equations as follows

$$\frac{d}{dt}a = -\left(\frac{\gamma_a}{2} + i\Delta_a\right)a - ig_0b + \varepsilon_c + \sqrt{\gamma_a}a_{\text{in}}, \quad (3)$$

$$\frac{d}{dt}b = -\left[\frac{\gamma_b}{2} + i(\Delta_b + g_b q)\right]b - ig_0a + \sqrt{\gamma_b}b_{\text{in}}, \quad (4)$$

$$\frac{d}{dt}q = \omega_m p, \quad (5)$$

$$\frac{d}{dt}p = -\omega_m q - g_b b^\dagger b - \frac{\gamma_m}{2}p + \xi, \quad (6)$$

where γ_a, γ_b and γ_m are the damping rates of cavity A, cavity B, and the moving mirror, respectively. $q = \frac{1}{\sqrt{2}}(c + c^\dagger)$, $p = \frac{1}{i\sqrt{2}}(c - c^\dagger)$, and $g_b = \sqrt{2}G$. ξ is a Brownian stochastic force with zero mean value, i.e. $\langle \xi(t) \rangle = 0$, which comes from the coupling of the oscillating mechanical resonator to its thermal environment and satisfies correlation [40–43]

$$\langle \xi(t) \xi(t') \rangle = \frac{\gamma_m}{2\omega_m} \int \frac{d\omega}{2\pi} \omega e^{-i\omega(t-t')} \left[1 + \coth\left(\frac{\hbar\omega}{2k_B T}\right) \right], \quad (7)$$

where k_B is the Boltzmann constant and T is the effective temperature of the environment of the mechanical resonator. a_{in} and b_{in} represent the vacuum radiation noises input to the cavity A and B with $\langle a_{\text{in}}(t) \rangle = \langle b_{\text{in}}(t) \rangle = 0$, and they obey the following correlation functions [44]

$$\langle a_{\text{in}}^\dagger(t) a_{\text{in}}(t') \rangle = 0, \quad (8)$$

$$\langle a_{\text{in}}(t) a_{\text{in}}^\dagger(t') \rangle = \delta(t - t'), \quad (9)$$

$$\langle b_{\text{in}}^\dagger(t) b_{\text{in}}(t') \rangle = 0, \quad (10)$$

$$\langle b_{\text{in}}(t) b_{\text{in}}^\dagger(t') \rangle = \delta(t - t'). \quad (11)$$

Here, we have assumed that whole system is in a low temperature environment, and therefore the equilibrium mean thermal photon numbers in two cavities at optical frequencies have been neglected.

The dynamic of the system is usually determined by the small fluctuations when the system reaches the steady-state. Thus, let us now apply semiclassical approximation to solve the steady-state with small quantum fluctuations. That is, we assume $a = \alpha_0 + \delta a$, $b = \beta_0 + \delta b$, $q = q_0 + \delta q$, here α_0, β_0 and q_0 are the mean values of the cavity fields and mechanical mode when the system reaches steady-states, and operators $\delta a, \delta b$ and δq describe the small fluctuations around steady-state and satisfy the condition $\langle \delta a \rangle = 0$, $\langle \delta b \rangle = 0$ and $\langle \delta q \rangle = 0$. The steady-state values satisfy the following equations

$$\left(\frac{\gamma_a}{2} + i\Delta_a\right)\alpha_0 + ig_0\beta_0 = \varepsilon_c, \quad (12)$$

$$\left[\frac{\gamma_b}{2} + i(\Delta_b + g_b q_0)\right]\beta_0 + ig_0\alpha_0 = 0, \quad (13)$$

$$\omega_m q_0 = -g_b |\beta_0|^2. \quad (14)$$

Here, we have used the factorization assumption, e.g., $\langle qb \rangle =$

$\langle q \rangle \langle b \rangle$. The dynamics of small fluctuations around steady-state can be described by linearizing Eqs. (3-6) as

$$\frac{d}{dt} \delta a = -\left(\frac{\gamma_a}{2} + i\Delta_a\right) \delta a - ig_0 \delta b + \sqrt{\gamma_a} a_{in}, \quad (15)$$

$$\begin{aligned} \frac{d}{dt} \delta b = & -\left[\frac{\gamma_b}{2} + i(\Delta_b + g_b q_0)\right] \delta b - ig_b \beta_0 \delta q \\ & - ig_0 \delta a + \sqrt{\gamma_b} b_{in}, \end{aligned} \quad (16)$$

$$\frac{d}{dt} \delta q = \omega_m \delta p, \quad (17)$$

$$\frac{d}{dt} \delta p = -\omega_m \delta q - g_b (\beta_0^* \delta b + \beta_0 \delta b^\dagger) - \frac{\gamma_m}{2} \delta p + \xi \quad (18)$$

here, the high order terms of small fluctuations, e.g., $\delta q \delta a$, have been neglected. The system is stable only if all the eigenvalues of the coefficient matrix of the above differential equations have negative real parts, and the stability condition can be given explicitly by using the Routh-Hurwitz criterion [45]. However, it is too cumbersome to be given here. All the parameters we will use satisfy the stability condition, and it is easy to fulfill it for the driven field in our system is weak.

By applying the Fourier transform and solving dynamical equations in the frequency domain, we can obtain

$$\begin{aligned} \delta a(\omega) = & E(\omega) a_{in}(\omega) + F(\omega) a_{in}^\dagger(\omega) \\ & + G(\omega) b_{in}(\omega) + H(\omega) b_{in}^\dagger(\omega) \\ & + Q(\omega) \xi(\omega), \end{aligned} \quad (19)$$

where

$$E(\omega) = \sqrt{\gamma_a} \frac{A_{11}(\omega)}{D(\omega)}, \quad (20)$$

$$F(\omega) = -\sqrt{\gamma_a} \frac{A_{22}(\omega)}{D(\omega)}, \quad (21)$$

$$G(\omega) = \sqrt{\gamma_b} \frac{A_{33}(\omega)}{D(\omega)}, \quad (22)$$

$$H(\omega) = -\sqrt{\gamma_b} \frac{A_{44}(\omega)}{D(\omega)}, \quad (23)$$

$$Q(\omega) = -i \frac{g_b \chi(\omega)}{\omega_m D(\omega)} [\beta_0 A_{33}(\omega) + \beta_0^* A_{44}(\omega)], \quad (24)$$

and

$$\begin{aligned} A_{11}(\omega) = & \left[\frac{\gamma_a}{2} - i(\Delta_a + \omega)\right] \left[\left(\frac{\gamma_b}{2} - i\omega\right)^2 + \Delta_b'^2\right] \\ & - \left[\frac{\gamma_a}{2} - i(\Delta_a + \omega)\right] g_b^4 |\beta_0|^4 \left(\frac{\chi(\omega)}{\omega_m}\right)^2 \\ & + g_0^2 \left[\frac{\gamma_b}{2} + i(\Delta_b' - \omega)\right], \end{aligned} \quad (25)$$

$$A_{22}(\omega) = -ig_0^2 g_b^2 (\beta_0)^2 \frac{\chi(\omega)}{\omega_m}, \quad (26)$$

$$\begin{aligned} A_{33}(\omega) = & -ig_0 \left[\frac{\gamma_a}{2} - i(\Delta_a + \omega)\right] \left[\frac{\gamma_b}{2} - i(\Delta_b' + \omega)\right] \\ & - ig_0^3, \end{aligned} \quad (27)$$

$$A_{44}(\omega) = -g_0 g_b^2 (\beta_0)^2 \frac{\chi(\omega)}{\omega_m} \left[\frac{\gamma_a}{2} - i(\Delta_a + \omega)\right], \quad (28)$$

$$D(\omega) = \left[\frac{\gamma_a}{2} + i(\Delta_a - \omega)\right] A_{11}(\omega) + ig_0 A_{33}(\omega), \quad (29)$$

here, we introduce $\Delta_b' = \Delta_b + g_b q_0 - g_b^2 |\beta_0|^2 \frac{\chi(\omega)}{\omega_m}$, and the dynamical response function of the mirror

$$\chi(\omega) = \frac{\omega_m^2}{(\omega_m^2 - \omega^2 - i\omega\gamma_m/2)} \quad (30)$$

with $\chi^*(\omega) = \chi(-\omega)$.

IV. SECOND-ORDER CORRELATION FUNCTIONS AND STATISTICAL PROPERTIES OF CAVITY FIELD

In this section, we are going to investigate photon statistics of cavity A by using the analytical expression given in the last section. By taking $a = \alpha_0 + \delta a$, the equal-time second-order correlation function $g_{aa}^{(2)}(0)$ of the light field in the cavity A can be given as

$$\begin{aligned} g_{aa}^{(2)}(0) = & \frac{|\alpha_0|^4 + 4|\alpha_0|^2 \langle \delta a^\dagger(t) \delta a(t) \rangle}{\left(|\alpha_0|^2 + \langle \delta a^\dagger(t) \delta a(t) \rangle\right)^2} \\ & + \frac{2\text{Re}[(\alpha_0^*)^2 \langle \delta a(t) \delta a(t) \rangle]}{\left(|\alpha_0|^2 + \langle \delta a^\dagger(t) \delta a(t) \rangle\right)^2} \\ & + \frac{|\langle \delta a(t) \delta a(t) \rangle|^2 + 2\langle \delta a^\dagger(t) \delta a(t) \rangle^2}{\left(|\alpha_0|^2 + \langle \delta a^\dagger(t) \delta a(t) \rangle\right)^2}, \end{aligned} \quad (31)$$

where the third term comes from the four-operator correlation by using properties of the Gaussian process [40, 41], and the second term is only one which may take negative value. Using the expression of $\delta a(t)$, Eqs. (19), and taking the correlations Eqs. (7-11), the correlation of $\delta a(t)$ and $\delta a^\dagger(t)$ are given as

$$\langle \delta a^\dagger(t) \delta a(t) \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega X_{a^\dagger a}(\omega), \quad (32)$$

$$\langle \delta a(t) \delta a(t) \rangle = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega X_{aa}(\omega), \quad (33)$$

where

$$\begin{aligned} X_{a^\dagger a}(\omega) = & |Q(-\omega)|^2 \frac{\gamma_m}{2\omega_m} \omega \left[1 + \coth\left(\frac{\hbar\omega}{2k_B T}\right)\right] \\ & + |F(-\omega)|^2 + |H(-\omega)|^2, \end{aligned} \quad (34)$$

$$\begin{aligned} X_{aa}(\omega) = & Q(\omega) Q(-\omega) \frac{\gamma_m}{2\omega_m} \omega \left[1 + \coth\left(\frac{\hbar\omega}{2k_B T}\right)\right] \\ & + E(\omega) F(-\omega) + G(\omega) H(-\omega). \end{aligned} \quad (35)$$

For comparison, before analyzing the statistical properties of light field in cavity A , we show the equal-time second-order correlation function $g^{(2)}(0)$ of the standard optomechanical system in Fig. 2(a). The standard optomechanical system consists of a cavity with an oscillating mirror at the one end [8, 9]. Figure 2(a) shows that strong antibunching appears in standard optomechanical system under strong optomechanical coupling condition $G > \gamma$, and this agrees with Ref. [8].

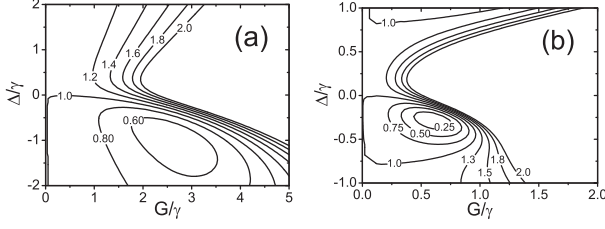


FIG. 2: (a) Equal-time second order correlation function of standard optomechanical system $g^{(2)}(0)$ plotted as functions of Δ and G . (b) Equal-time second order correlation function of cavity A , $g_{aa}^{(2)}(0)$ given by Eq. (31), plotted as functions of Δ and G for $g_0 = 3\gamma$. The other parameters are $\gamma_a = \gamma_b = \gamma$, $\varepsilon_c = 0.01\gamma$, $\omega_m = 10\gamma$, $\omega_m/\gamma_m = 1000$, and $T \approx 0$ K.

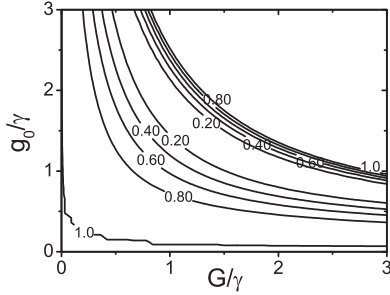


FIG. 3: Equal-time second order correlation function of cavity A , $g_{aa}^{(2)}(0)$ given by Eq. (31), plotted as functions of both g_0 and G for $\Delta = -0.24\gamma$. The other parameters are the same as in Fig. 2 (b).

Now, let us focus on the statistical properties of light field in cavity A . From now on, we assume that the two cavities (cavity A and B) are the same as each other, that is, $\gamma_a = \gamma_b = \gamma$, $\omega_a = \omega_b$, the detunings are denoted by $\Delta_a = \Delta_b \equiv \Delta$, and normalize all the parameters to γ . The equal-time second-order correlation function of the photons inside cavity A , $g_{aa}^{(2)}(0)$, is given in Fig. 2(b). We observe that strong antibunching appears near the point for $\Delta = -0.24\gamma$ and $G = 0.6\gamma$ in cavity A as $g_0 = 3\gamma$. That is, the photons in the cavity can exhibit strong antibunching when it is coupled to an optomechanical system under weak condition $G < \gamma$.

The optomechanical interaction in cavity B and the quantum interference effect between the two cavities (cavity A and cavity B) are responsible for the photon antibunching effect [7]. The interference is between two paths for two-photon excitation in cavity A : (a) the direct excitation from one photon to two photons inside the cavity; and (b) one photon tunneling from cavity A to cavity B , then exciting another photon in cavity A , and finally the photon inside cavity B tunneling back to cavity A . Thus the destructive interference between two paths reduces the probability of two-photon excitation in cavity A .

Two-dimensional plot of the equal-time second-order correlation function of cavity A , $g_{aa}^{(2)}(0)$, as functions of both g_0 and G is shown in Fig. 3 for $\Delta = -0.24\gamma$. With the increas-

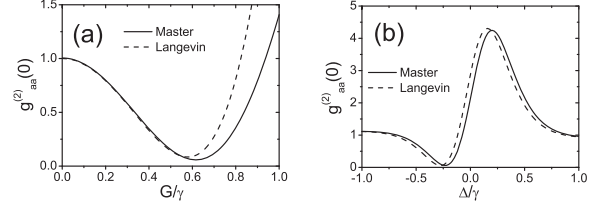


FIG. 4: (a) Equal-time second order correlation function of cavity A , $g_{aa}^{(2)}(0)$, plotted as functions of G for $\Delta = -0.24\gamma$. (b) Dependence of $g_{aa}^{(2)}(0)$ on Δ for $G = 0.6\gamma$. The solid curves are calculated by master equation, given by Eq. (38), and the dashed curves are obtained from quantum Langevin equations, given by Eq. (31). The other parameters are the same as in Fig. 2 (b).

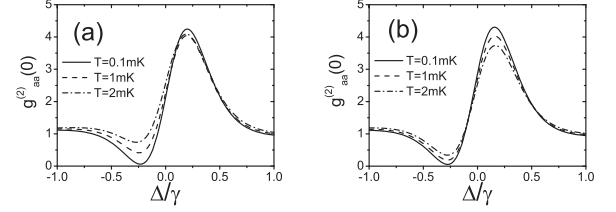


FIG. 5: Equal-time second order correlation function of cavity A , $g_{aa}^{(2)}(0)$, plotted as functions of Δ for different temperatures at $G = 0.6\gamma$ and $\gamma/2\pi = 1$ MHz. (a) The results are obtained by master equation approach, given by Eq. (38), (b) the results are obtained by quantum Langevin equations method, given by Eq. (31). The other parameters are the same as in Fig. 2 (b).

ing of g_0 , the value of G for getting the strong antibunching $g_{aa}^{(2)}(0)$ is drawn back from the strong optomechanical interaction condition to the weak optomechanical coupling domain. This implies that destructive quantum interference effect in the coupled system can be used to lower the strength of the optomechanical interaction that is required to achieve strong antibunching.

V. NUMERICAL SOLUTION BY MASTER EQUATION

In this part, we calculate the second-order correlation function by numerically solving the master equation of the density matrix, and compare the results to the predictions given by analytical solution derived above. The master equation of the coupled system is given as [46]

$$\begin{aligned} \frac{\partial \rho}{\partial t} = & \frac{1}{i\hbar} [\tilde{H}, \rho] + \frac{\gamma_a}{2} (2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a) \\ & + \frac{\gamma_b}{2} (2b\rho b^\dagger - b^\dagger b \rho - \rho b^\dagger b) \\ & + \frac{\gamma_m}{2} (2c\rho c^\dagger - c^\dagger c \rho - \rho c^\dagger c) \\ & + \gamma_m \bar{n}_m (c\rho c^\dagger + c^\dagger \rho c - c^\dagger c \rho - \rho c^\dagger c), \end{aligned} \quad (36)$$

where \bar{n}_m is the mean thermal phonon number of the moving mirror given by the Bose-Einstein statistics $\bar{n}_m =$

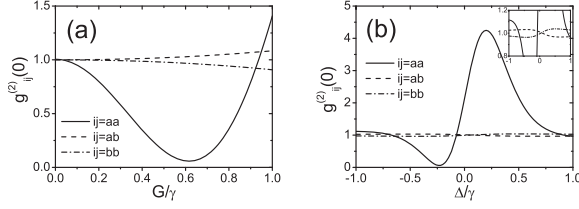


FIG. 6: (a) Equal-time second order correlation function $g_{ij}^{(2)}(0)$ plotted as functions of G at $\Delta = -0.24\gamma$, $ij = aa$ for cavity A (solid), $ij = ab$ for the cross correlation between the two cavities (dashed), and $ij = bb$ for cavity B (dashed-dot). (b) Dependence of $g_{ij}^{(2)}(0)$ on Δ for $G = 0.6\gamma$. The other parameters are the same as in Fig. 2 (b).

$[\exp(\hbar\omega_m/k_B T) - 1]^{-1}$, k_B is the Boltzmann constant and T is the effective temperature of the moving mirror. The master equation can be solved in the basis of the photon and phonon number states $|n_a, n_b, n_m\rangle$, and ρ can be written as density matrix

$$\rho = \rho_{n_a, n_b, n_m; n'_a, n'_b, n'_m}(t) |n_a, n_b, n_m\rangle \langle n'_a, n'_b, n'_m|. \quad (37)$$

If the elements of the steady state density matrix, $\rho_{n_a, n_b, n_m; n'_a, n'_b, n'_m}$, are given, the equal-time second-order correlation function can be easily calculated as

$$g_{aa}^{(2)}(0) = \frac{\text{Tr}[\rho a^{\dagger 2} a^2]}{[\text{Tr}(\rho a^{\dagger} a)]^2}, \quad (38)$$

$$g_{bb}^{(2)}(0) = \frac{\text{Tr}[\rho b^{\dagger 2} b^2]}{[\text{Tr}(\rho b^{\dagger} b)]^2}, \quad (39)$$

$$g_{ab}^{(2)}(0) = \frac{\text{Tr}[\rho a^{\dagger} b^{\dagger} b a]}{[\text{Tr}(\rho a^{\dagger} a)][\text{Tr}(\rho b^{\dagger} b)]}, \quad (40)$$

where $g_{ab}^{(2)}(0)$ is the cross correlation between the photons in cavity A and B.

For comparison, the second order correlation functions calculated by the master equation and quantum Langevin equations are shown in the same figure as functions of G for $\Delta = -0.24\gamma$ in Fig. 4 (a), and as functions of Δ for $G = 0.6\gamma$ in Fig. 4 (b). From Fig. 4 (a), we can see that the results obtained by the two methods agree with each other as G is small, especially for $G < 0.6\gamma$. As shown in Fig. 4 (b), for $G = 0.6\gamma$, the predictions by the two methods match quantitatively. But with the further increasing of G , the difference between them becomes significant, and linearized quantum Langevin equations method can only describe this qualitatively.

The second order correlation functions calculated by the master equation and quantum Langevin equations for different temperatures are shown in Fig. 5. From Fig. 5 we can see that the increase of the phonons will suppress the exhibition of

antibunching effect. In order to get strong antibunching, the mean phonon number of the environment must be small because the phonons in the environment may disturb the quantum statistics of the system. Compare the results in Fig. 5 (a) and (b), in low temperature ($T = 0.1$ mK), the results obtained by the two methods are close; as the temperature goes higher ($T = 2$ mK), the discrepancy between them becomes larger.

Finally, let us give a glance of the statistical properties of photons in the entire system. The equal-time second order correlation functions $g_{ij}^{(2)}(0)$ can be calculated by using Eqs. (38-40) and the results are shown in Fig. 6. From Fig. 6 (a) we can see that, under weak optomechanical interaction condition ($G < \gamma$), there is strong antibunching in cavity A around $G = 0.6\gamma$, while weak antibunching in cavity B and bunching for the photons between the two cavities.

Dependence of $g_{ij}^{(2)}(0)$ on Δ is drawn in Fig. 6 (b). Fig. 6 (b) shows us two interesting phenomena as Δ is in different domains: For $\Delta < -0.08\gamma$, there is strong antibunching in cavity A and weak antibunching in cavity B, while the cross correlation between the modes in the two cavities exhibits bunching. On the contrary, when $\Delta > 0.1\gamma$, there is bunching in cavity A and cavity B, while the cross correlation between the modes in the two cavities exhibits weak antibunching $g_{ab}^{(2)}(0) < 1$. Under weakly driven condition [37], $g_{aa}^{(2)}(0) < 1$ and $g_{bb}^{(2)}(0) < 1$ indicate that there is no more than one photon in each cavity, and $g_{ab}^{(2)}(0) > 1$ shows that there is big chance that each cavity has one photon simultaneously. $g_{aa}^{(2)}(0) > 1$ and $g_{bb}^{(2)}(0) > 1$ indicate that there is big chance for more than one photon present in each cavity, while $g_{ab}^{(2)}(0) < 1$ shows that the probability that each cavity has one photon simultaneously is low. In other words, when the system is driven weakly, and there are two photons in the coupled system, if $\Delta < -0.08\gamma$, they are likely to be in the state that each cavity has one photon simultaneously, and if $\Delta > 0.1\gamma$, they prefer to stay in one of the cavities together at the same time.

VI. CONCLUSIONS

We have studied the photon statistics of a cavity linearly coupled to an optomechanical system. Due to destructive quantum interference effect between different paths for two-photon excitation, the cavity can exhibit strong photon antibunching with weak optomechanical interaction in the optomechanical system. Both analytical and numerical methods are employed to figure out our result. The result brings hope to us of observing blockade effect with current experimental parameters of optomechanics.

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